# Early Modern Mathematical Instruments

*By Jim Bennett**

**ABSTRACT**

In considering the appropriate use of the terms "science" and "scientific instrument," tracing the history of "mathematical instruments" in the early modern period is offered as an illuminating alternative to the historian's natural instinct to follow the guiding lights of originality and innovation, even if the trail transgresses contemporary boundaries. The mathematical instrument was a well-defined category, shared across the academic, artisanal, and commercial aspects of instrumentation, and its narrative from the sixteenth to the eighteenth century was largely independent from other classes of device, in a period when a "scientific" instrument was unheard of.

FOR THE PURPOSE OF REVIEWING THE HISTORY of mathematical instruments and the place the subject might command in the history of science, if we take "early modern" to cover the period from the sixteenth to the eighteenth century, a first impression may well be one of a change from vigorous development in the sixteenth century to relatively mundane stability in the eighteenth. More careful scrutiny suggests that this perception is relative and depends more on our priorities as historians than on the interests of the instrument manufacturers or users. In fact, the role of mathematical instruments and the agenda of their designers and makers were fairly steady over the period, and what changes is how these might be viewed in the broader context of natural philosophy and its instruments. Following the mathematical thread in instrumentation consistently through this time has not been a popular option, even for instrument historians and even though it has a demonstrable, categorical presence. It is all too easy to be diverted by telescopes, microscopes, barometers, air-pumps, electrical machines, and the like, once these seductive newcomers arrive on the scene, parading their novel and challenging entanglement with natural knowledge. Yet, as a category, "mathematical instruments" retained its currency and meaning throughout the period, while commercial, professional, and bureaucratic engagement with such objects continued to expand. It remained more common to want to know the time than the barometric pressure, and, despite the growing popularity of orreries, there was still more money in octants.

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When historians encounter mathematical instruments, such as astrolabes, sundials, quadrants, surveyors' theodolites, or gunners' sights and rules, it will generally not be in the secondary histories where they first learn their trade but, instead, in museums. Coming from book-learned history, with its recent material turn in the history of science, and looking for a material culture from the sixteenth and seventeenth centuries can be a dispiriting and perplexing experience. Collections rich in material from the period present uncompromising rows of instruments that are clearly challenging in their technical content but seem obsessed with the wrong kinds of questions. Sundials are plentiful, while telescopes and microscopes are fabulously rare. Horary quadrants and gunners' sights present themselves in baffling varieties, but not a single air-pump seems to survive.

The dominant instrument culture up to the end of the seventeenth century characterized itself as "mathematical." By then it incorporated a range of applications of mainly geometrical techniques to an array of what had become, at least in aspects of their practice, *mathematical* arts. Astronomy had set the pace for instrumentation. By the sixteenth century, instrumentation had long been an integral part of its practice—instruments for the immediate requirement of positional measurement, leading on to those with more particular functionality, such as astrolabes, sundials, and horary quadrants. The armillary sphere (see Figure 1) may well have begun as a measuring instrument, but because its arrangement of circles reflected those used by astronomers for registering the heavenly motions, it could also be used for teaching the practice of astronomy and for a limited amount of calculation.

The armillary sphere perfectly indicates the ambiguity of some mathematical instruments, in turn reflecting tensions in the discipline of astronomy itself: are the circles and motions of the instrument intended primarily to correspond to the heavens or to the geometrical practice of astronomers? Many books in the "sphaera" tradition, popularized in the work of Sacrobosco, can be read equally easily with reference to the sky or to an artificial instrument, the armillary sphere. The emergence of instruments as a prominent instantiation of disciplinary practice may be more profound than has generally been allowed, to be acknowledged alongside instantiation through treatises, terminologies, diagrams, constructions, and rules. An important character of mathematical instruments is that they face more toward disciplinary practice than toward the natural world. That is where they derive their regulation and legitimacy, while the discipline in turn is partly characterized through its engagement with artificial instruments.

In seeking to characterize the role of instruments in disciplinary practice, we can look at how they appear in published treatises. Descriptions are found within more general astronomical works, when some account is needed of the nature of measurement, and an important example of this would be Ptolemy's *Almagest* itself. But even before the first printed edition (1515) of *Almagest*, an independent literature of printed treatises on mathematical instruments was launched in 1513 by Johann Stoeffler with his *Elucidatio fabricae ususque astrolabii.*[1] The formula of "construction and use" was adopted for many of the accounts of particular mathematical instruments that were published through the century. Stoeffler first tells his readers how to make an astrolabe, then, in a series of worked examples, how to use it. The treatise presents itself not in the context of an overarching discipline but as an independent manual for a particular instrument.

Through the sixteenth century many accounts of individual instruments were published,

[1] Johann Stoeffler, *Elucidatio fabricae ususque astrolabii* (Oppenheim, 1513).

***Figure 1.*** *Armillary sphere by Carlo Plato, Rome, 1598. (Museum of the History of Science, inventory no. 45453.)*

alongside treatises on sets or selections of instruments, whether closely related in type, such as in Sebastian Münster's books on sundials, or covering as broad a range as possible, as in Giovanni Paolo Gallucci's *Della fabrica et uso di diversi stromenti di astronomia et cosmographia* (1597).[2] This relative detachment from what might be considered the parent or foundational discipline, seen in the publication of books devoted solely to instruments and their use, is worth a moment's thought. Instrument development had a narrative that ran alongside the parent discipline but was not dependent on changes or advances in astronomy. The originality of astronomers and other mathematicians could be exercised through the design and improvement of instruments; a network of instrumental relations could spread among separate disciplines (astronomy, navigation, surveying, architecture, warfare) based on practical techniques deployed in instruments in different fields of practice; a community of makers promoted the development of these links and of instrument design and use as a commercial imperative, while a growing number of practitioners did the same in the hope of professional advantage. In short, instruments had a life that was not entirely dependent on an academic mathematical discipline, and this is reflected in the range of sixteenth-century publications.

At the end of the century, the rise of the independent treatise on one or more mathematical instruments reflects back into astronomical measurement itself through the publication of Tycho Brahe's *Astronomiae instauratae mechanica* of 1598.[3] This detailed account of his observatory and its individual instruments, set out one after another in a thorough and comprehensive manner, established an influential precedent for ambitious observatory astronomers; later examples in our period include Johannes Hevelius, John Flamsteed, and Ole Roemer.

Before moving the narrative into the seventeenth century, and as a narrative thread to lead us there, we might briefly consider the subject of dialing and the division of instruments—sundials and horary quadrants—that characterized the subject. If there is a mismatch between the interests of early modernists and those of their actors with regard to mathematical instruments generally, the disparity is even greater when it comes to dialing. Not only might the problem of finding time from the sun, mostly with portable rather than fixed instruments, be solved in a great variety of imaginative and technically challenging ways, be applicable to different conventions of dividing the day or night and numbering the hours, be made relevant to the whole earth instead of a single latitude, and so on; the ambitions of such instruments could extend far beyond telling the time "here and now"—or even "there and then." Their functionality could extend outside time telling into other areas of astronomical problem solving.

In the sixteenth century dialing was a vibrant, challenging, even competitive astronomical and geometrical discipline, with a large following, many new designs of instruments, and a correspondingly healthy output of publications. It was related closely to the popular contemporary discipline of cosmography—the mathematical, or part-mathematical, discipline dealing with the geometrical relationships between the heavens and the earth. Leading mathematicians, such as Regiomontanus, Peter Apianus, Gemma Frisius, and Oronce Fine, were centrally involved, seeing dialing as an integral part of their disciplin-

[2] Sebastian Münster, *Compositio horologiorum* (Basel, 1531); Münster, *Horologiographia* (Basel, 1533); and Giovanni Paolo Gallucci, *Della fabrica et uso di diversi stromenti di astronomia et cosmographia* (Venice, 1597).

[3] Tycho Brahe, *Astronomiae instauratae mechanica* (Wandsbeck, 1598).

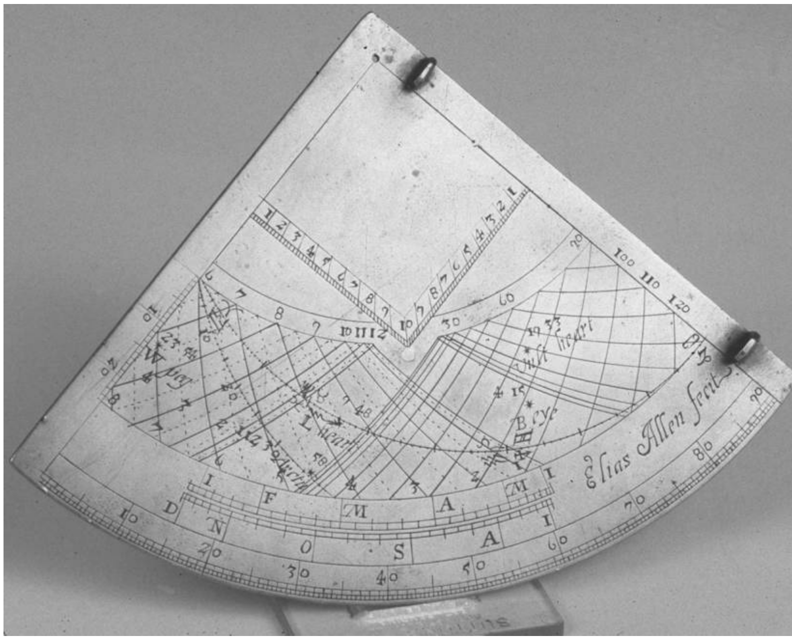


***Figure 2.*** *Gunter quadrant by Elias Allen, circa 1630. (Whipple Museum of the History of Science, Wh. 1764.)*

ary practice.[4] Yet dialing is now all but ignored by historians of science. Extending our appreciation to this aspect of contemporary astronomy will broaden our grasp of its scope and methodology. Planetary theory is important, of course, but to achieve a rounded account of the discipline we cannot afford to neglect an aspect that was clearly significant for many of its practitioners.

Following the development of dials and quadrants takes us seamlessly into the seventeenth century and the rise in importance of English mathematicians and makers. The institutional mission of Gresham College in London, with its recognition of the practical mathematical arts and sciences as a subject for the kind of systematic treatment that might underpin a series of weekly lectures, was important for bringing the English into a European movement—one that combined learning, technical innovation, practical application, publication, manufacture, and commerce. The professor of astronomy from 1619 to 1626, Edmund Gunter, had a particular commitment to, and success with, instrumentation, and his eponymous quadrant (see Figure 2) established itself as a standard portable astronomical instrument through the seventeenth and eighteenth centuries.[5] Characteristic of the sundial's functionality being extended beyond time telling, by restricting his projection of hour lines to a particular latitude Gunter was able to include other lines, such as the horizon, the ecliptic, and lines of solar azimuth, so that his quadrant could be

[4] Jim Bennett, "Sundials and the Rise and Decline of Cosmography in the Long Sixteenth Century," *Bulletin of the Scientific Instrument Society*, 2009, no. 101, pp. 4–9. More generally, see Hester Higton, *Sundials at Greenwich* (Oxford: Oxford Univ. Press, 2002).

[5] Edmund Gunter, *The Description and Use of the Sector, the Crosse-Staffe, and Other Instruments* (London, 1624).

applied to a range of astronomical calculations. Gunter worked closely with the instrument maker Elias Allen, whose shop near St. Clement's Church in the Strand was a place of resort and exchange for many interested in practical mathematics.

Another of Allen's mathematical patrons was William Oughtred, whose design of the universal equinoctial ring dial, like the Gunter quadrant, was popular through the eighteenth century. Heir to the dialing literature from sixteenth-century Continental mathematicians, Oughtred's design was developed from the astronomer's rings of Gemma Frisius. A design of his own was his "horizontal instrument," a dial based on a projection onto the horizon of the sun's daily paths throughout the year, the ecliptic, and the equator; like Gunter's quadrant, it had a range of astronomical functionality. There were fixed and portable versions, and the latter were combined with one of the most important achievements of the new English school of mathematical instrumentation, the delivery of logarithmic calculation as a practical technique by means of an instrument. The close links between publication and instrument manufacture are maintained: the horizontal instrument and the "circles of proportion" (Oughtred's circular logarithmic rule) occupied either side of a portable instrument (to be had in the shop of Elias Allen) and appeared together in Oughtred's book of 1632 (published and sold by Allen).[6]

The development of specialized or professionalized calculation around this time is an instructive episode for understanding the nature of mathematical instruments. Imperatives from the world beyond mathematics—in warfare and navigation—were creating opportunities for ambitious mathematicians, adaptable practitioners, and entrepreneurial makers. The kinds of proportional calculations required of the gunner in working out the weight of shot or the appropriate charge of powder were amenable to geometrical handling using similar triangles and could readily be rendered instrumental by a "sector" (or "compass of proportion"), where a pair of scales on the faces of flat rules connected by a compass joint could be opened to different angles, according to the proportionality required. Galileo's "geometrical and military compass" is only the best known of a range of such instruments, where combinations of different pairs of lines might handle not just the direct proportions needed, say, by surveyors drawing plans to scale, but also lines for areas (useful again to surveyors as well as carpenters) and volumes (used by, for example, masons, architects, or gunners).[7]

The sector embodied a very adaptable technique, and, after the mathematics of the Mercator chart for sailing had been explained by another mathematician in the Gresham circle, Edward Wright, Gunter designed a specialized version for managing the kind of trigonometrical calculations required when working with a chart where the scale varied as the secant of the latitude. Gunter went on to devise a rule with lines carrying log-trig functions, so that multiplications and divisions could be carried out by the straightforward addition or subtraction of lengths transferred by means of a pair of dividers. Two such rules (dispensing with the dividers) created the once-familiar logarithmic slide rule. Logarithms had only recently been invented by John Napier, and their rapid application in Gunter's rule and Oughtred's circles indicates the continuing significance of the role played by instruments in contemporary mathematics.

The practical discipline that had introduced such technical novelties as these maintained a strong commercial presence through the seventeenth century, now with the addition of

[6] William Oughtred, *The Circles of Proportion and the Horizontall Instrument* (London, 1632).

[7] Filippo Camerota, *Il compasso di Fabrizio Mordente, per la storia del compasso di proporzione* (Florence: Olschki, 2000).

English workshops to those of continental Europe. Indeed, as latecomers to the field, the English (which means largely London) makers seem particularly active. It is worth stressing that mathematical instrument makers were specialist craftsmen who formed an identifiable and readily understood trade. In some Continental cities there was a measure of guild regulation, but in London, although the makers had to belong to some London company, it did not matter which, and there was no nominated home for these mathematical workers. So, they are found in the Grocers, the Drapers, and many others. In the seventeenth century the arts of war become somewhat less evident in the literature and surveying somewhat more so; the two fields are linked in the technique of triangulation and the instruments designed to facilitate it. In surveying these are used for drawing maps; in a military context they are range finders.

Mathematical instrument makers did not become involved in the production of telescopes or microscopes. Makers of optical instruments, if they were not astronomers themselves turning their hands to practical work, emerged from among the most skilled and ambitious of the spectacle makers, while mathematical instrument makers continued their independent and customary trade, applying their engraving skills to practical ends in the mathematical arts. They were not concerned with discoveries in the far away or the very small. But there was one area of practical optics that came to impinge on their world—namely, in measuring instruments for astronomy, where telescopic sights were applied to divided instruments to increase the accuracy of the alignment of the index on the distant target. In a superficial sense this development witnessed a connection between the two separate areas of work: an optical instrument was combined with an astronomical measuring instrument and the division between optical and mathematical had been breached. But the added telescope was intended simply to improve the accuracy of the measurement; it did not alter the fundamental function of the instrument. The same conjunctions were made in surveying and navigation, with the addition of telescopic sights to designs of theodolite and sextant, but more effectively in the eighteenth century.

Retailing (if not manufacturing) across the division between mathematical and optical instruments came in a more profound way toward the end of the seventeenth century, early instances being found in the practices of the London makers Edmund Culpeper and John Rowley. It was here that the relatively loose regulation of the City companies in controlling manufacturing boundaries was a significant advantage over more restrictive regimes. So we enter the eighteenth century with a commercial if not a conceptual link between the two areas of instrumentation, and, as the popularity of experimental natural philosophy grew, a further commercial opportunity arose through the potential for a trade in the apparatus of natural philosophy, such as air-pumps, electrical machines, and all the burgeoning content of the "cabinet of physics."

Much of the staple fare of the mathematical instrument maker remained in his repertoire through the eighteenth century, while again new designs were added. Sundials and quadrants were as popular as ever—before being tempted to add "despite increasing numbers of pocket watches," remember that watches had to be set to time. Though with roots in designs by Hooke and Newton, angular measurement by the principle of reflection now found original and successful applications in octants, sextants, and circles, used in navigation and surveying, while more ambitious theodolites, perhaps adding an altitude arc or circle and a telescope to the azimuth function (see Figure 3), made a significant impression on the practice of surveyors. Professional mathematical practitioners such as these came to be associated even more than previously with their instrumentation, while the numbers of mathematical navigators and surveyors continued to grow. An informal

***Figure 3.*** *Theodolite by George Adams, London, late eighteenth century. (Museum of the History of Science, inventory no. 71425.)*

professionalization was under way, not regulated by institutes but encouraged through academies, more often private than public, through textbooks, and through the efforts of instrument makers.

There are three general reasons why the trade in mathematical instruments comes back from the margins of vision and into the range of historians of science. One arises from the entrepreneurial efforts of a number of makers to deal across all classes of instrument: in mathematics, optics, and natural philosophy. Prominent examples in London would be George Adams and Benjamin Martin—neither primarily identified with mathematical instruments but both including such instruments in their comprehensive range. They were also prominent in the rise of public, commercial natural philosophy and the growing

fashion for attending lectures and buying books and instruments. This drew octants and theodolites into the same commercial project as orreries and microscopes.

Second, the leading mathematical instrument makers became responsible for building the major instruments in the growing number of astronomical observatories, whose chief concern was astronomical measurement. It was in the eighteenth century that it first became normal to turn to commercial makers with commissions for observatory instruments. The recipients of such commissions were, of necessity, the leading mathematical makers, and, for reasons of status if nothing else, they retained their "mathematical" identity. Status was particularly evident in London, where these elite makers were patronized by the Board of Longitude and the Royal Observatory and could become Fellows of the Royal Society and be published in the *Philosophical Transactions*. Toward the end of the century mathematical instrument makers such as Jesse Ramsden and Edward Troughton came to occupy positions of such respect and influence that they are bound to find their places in the history of the period's science, yet there are sectors signed "Ramsden" and sundials by "Troughton."[8]

Third, the growing intellectual investment in accuracy of observation and measurement in natural investigation could be realized only through the mechanical knowledge and skill that had built up through centuries and now lay in the hands and workshops of mathematical instrument makers.

While a greater preoccupation with optical and experimental instruments is understandable, historians should not neglect the mathematical. Without understanding that category, and the categorization of instrument making as a whole in the period, we cannot properly appreciate other, more compelling, aspects of the instrument narrative. Mathematical instruments have a much longer history than optical and natural philosophical instruments and, although their established existence contributes to the possibility of instruments with other ambitions, the later types pursue separate developments, in different workshops and areas of practice, fully merging only later in *post hoc* commercial convenience. It is from this commercial union that the "scientific instrument" will emerge. Further, it is the practical mathematical work that engages with the worlds of commerce, bureaucracy, war, and empire, all of which speaks to the breadth of the history of science as practiced today. Finally, if we want to bring the likes of Ramsden and Troughton into our mainstream narrative, as we surely must given the importance ascribed to them by their contemporaries, that can be done only with an appreciation of the disciplinary tradition they recognized and through which they understood and defined their own work—that of the mathematical instrument.

[8] Anita McConnell, *Jesse Ramsden (1735–1800): London's Leading Scientific Instrument Maker* (Aldershot: Ashgate, 2007).