
**Comment**

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One enjoys this paper by Davis and Haltiwanger for the three things it tries to accomplish: (1) it is explicit about microeconomic underpinnings for macroaggregate phenomena; (2) it goes out and gathers new evidence, specifically that beyond aggregate employment and unemployment statistics there is great turbulence in employment at the level of manufacturing establishments; and (3) it begins to set up explicit prototypes with these microunderpinnings, built up around the evidence. My best tribute to this work is to take seriously the prototypes that are suggested. I try to do this in three ways. First, I argue that the prototypes can be made more operational, that it is possible to compute entire solutions paths. Second, the prototypes can be made more realistic; crucial missing features can be added. Third, and related, I complain that the authors themselves do not take this class of models seriously enough. They shy away from an explicit analysis of policy, yet there are various key social issues that cry out for a research program that is not unrelated to that envisioned by the authors.

I begin by describing the first, basic prototype model of the paper, so that we have a clear picture of the economy envisioned by the authors. Basic computational issues can be addressed as well in this simple framework. Next, following the authors, I add labor supply, though the model here is an alternative envisioned by the authors but not analyzed by
them. Here, there are firm specific shocks to labor demand, not household specific shocks to labor supply. In either setup with labor supply the complication is to retain the “representative consumer” construct even though there is explicit diversity across firms or households. But this can be done in the space of fractions or lotteries. At the same time, that space facilitates computation. Finally, I show that the prototype economy with labor supply can accommodate information and incentive problems. This will lead to a discussion of some policy issues.

The basic state variable of the simplest prototype is $H_t$, the fraction of workers matched to high-productivity sites at the very beginning of date $t$. There is one worker per site, and output if produced there, assuming the high-productivity status is retained would be $Y_H$. Fraction $1 - H_t$ workers are matched with low-productivity sites at the very beginning of date $t$, and output if produced there, assuming the worker stays, would be $Y_L$. At the next instant of date $t$, though, fraction $\sigma t$ of the high-productivity sites become low-productivity sites (fraction $1 - \sigma t$ of the high-productivity sites remain high). This then forces a decision about $\theta_t$, the fraction of workers at low-productivity sites who are to abandon production and move onward to high-productivity sites, arriving at the very beginning of date $t+1$. All this notation can be understood, then, by law of motion of state variable $H_t$, namely,

$$H_{t+1} = (1 - \sigma_t)H_t + \theta_t[1 - H_t + \sigma_t H_t].$$

(1)

To retain feasibility there must be a shadow, “unused” high-productivity site for every low-productivity site in the model. That is, it must be feasible to reallocate all workers in low-productivity sites to as yet unused high-productivity sites. For example, imagine there are 10 high-productivity sites at the beginning of date $t$, and 15 low-productivity sites. Among the highs, three revert to low productivity in the next instant; these sites are, in effect, “reallocated” to the low-productivity sector, though the movement is in the sense of accounting, not locations. In the low-productivity sector itself, four sites are to be abandoned. The four released workers from these abandoned sites are destined for the “shadow” high sector, consisting now of 15 old shadow highs plus the new three shadow highs. Note that the model thus has a symmetric, “bad news, good news” aspect. Shocks $\sigma_t$ that turn high-productivity sites into low-productivity sites also create new high-
productivity opportunities elsewhere. Hence the term, "reallocating shocks" $\sigma_t$.

Each and every household in the economy maximizes a discounted time-separable utility function.

$\sum_{t=0}^{\infty} \beta^t A_t U(c_t) \Rightarrow \text{consumption (per capita)}$

aggregate shock

Here $A_t$ is an aggregate demand shock at date $t$; when it is high it adds to the utility of consumption $c_t$. Note that all households are identical in preferences $U(\cdot)$, shocks $A_t$, and discount rate $\beta$. Different households may have different names, but they are to be treated alike nonetheless. The task then is to find a symmetric Pareto optimum.

For per capita consumption $c_t$ to be feasible it must satisfy the resource constraint, that output from operational high-production sites and operational low-production sites sum to it, namely:

$$c_t = (1-\sigma_t)H_tY_h + [1-H_t+\sigma_t H_t](1-\theta_t)Y_L.$$  \hspace{1cm} (2)

producing high not moving so producing low

The prototype can thus be summarized by a functional equation:

$$V(H_t,\sigma_t,A_t) = \max_{\theta_t} \{AU(c_t) + \beta E[V[H_{t+1},\sigma_{t+1},A_{t+1}]]\}. \hspace{1cm} (3)$$

Utility is maximized by choice of $\theta_t$ at each date $t$, conditioned on the state variable $H_t$, reallocation shock $\sigma_t$, and aggregate shock $A_t$. Equation (2) can be substituted into $c_t$ at date $t$ and law of motion (1) for $H_{t+1}$ can be embedded into future $V(\cdot)$.

Davis and Haltiwanger do some comparative static exercises on this model, asking what happens at date $t$ (only) conditioned on shocks $\sigma_t$ and $A_t$. Outcomes from some of the experiments can be signed, but some cannot. The obvious suggestion, though, is to compute the full dynamic stochastic equilibrium. This can be done in two ways.

First, imagine that $H_t$ can take on a finite though large number of values. Also, let $\sigma_t$ and $\theta_t$ take on at most finite number of values as well, and suppose these are such that given a finite set of potential values of $H_t$, the set of values $H_{t+1}$ is the same set of potential values. This grid technique has been used successfully by Sargent (1979) in a different
context. In any event, with \( A_t \) finite as well, value function \( V(\cdot) \) is then a finite dimensional vector. One need only make an initial guess for \( V \) on the right-hand side of (3); solve the maximum problem in (3) for \( \theta_t \) given each \( H_t, \sigma_t, \) and \( A_t \) combination; substitute the maximized solution into the objective function of (3); solve for \( V \) on the left-hand side; and finally iterate with this as a new guess for \( V \) on the right-hand side. This method of computing the value function \( V \) converges, and at the converged solution the method will dictate a choice of \( \theta_t \) as a function of \( H_t, \sigma_t, \) and \( A_t \). This policy rule will be fully optimal for the explicit infinite horizon stochastic dynamic program.

An alternative technique has been pursued by Coleman (1987) in a different context. Imagine \( H_t \) can take on a continuum of values after all. Then go to first-order equation 7, p. 23.

\[
A_tU'(c_t) = \beta E[1 - \sigma_{t+1}](Y_{t+1}/Y_t)A_{t+1}U'(c_{t+1})|A_t, \sigma_t].
\] (4)

Take a guess for next period’s policy function by naming a value for \( \theta_{t+1} \) at each of a finite number of values for \( H_{t+1} \) and given \( \sigma_{t+1} \) and \( A_{t+1} \). Interpolation, connecting the dots as it were, describes a policy function over the entire range of \( H_{t+1} \). Now solve first-order condition (4) for each \( \sigma_t \) and \( A_t \) at each of the finite number of values for \( H_t \), finding the maximizing value of \( \theta_t \). This, with interpolation as above, gives a policy function for the next iteration. In other contexts, such as Coleman’s, this numerical technique converges fast and is not sensitive to the number of grid points of \( H_t \) used for interpolation.

The point is that after choosing parameters for utility functions, discount rates, shock process, and the like, one can simulate entire dynamic paths. One just takes random draws off the supposed stochastic processes for \( A_t \) and \( \sigma_t \) and substitutes these into the compound optimal policy function. With these one can generate all time series and thus get explicit vector autoregressions without the need for identifying assumptions. Innovations in the stochastic processes for \( \sigma_t \) and \( A_t \) are directly linked to innovations in all derived, economic variables. Innovation experiments can trace out all relevant dynamics. I confess to being very curious about what these paths would look like.

Having solutions in hand, however, would beg some further important issues. In particular, what are the key features of the model and of the data that one is trying to match. The model as it stands literally has only job destruction and new job creation, because labor is as yet inelastic. Related, people either work or search; employment in this broader sense is constant. Finally, the model has a strong persistence characteris-
tic: new high-productivity jobs are as likely to crash as old ones. I’m not sure this last feature is matched in the data. The first two features definitely are not.

Troubled by some of these features, Davis and Haltiwanger add labor supply to the model, with utility for leisure entering linearly and subject to a stochastic shock. Here, let us take a somewhat different route, allowing a (common) concave nonseparable utility function for consumption and leisure but supposing output in each plant is random, even across plants in the high- (or low-) productivity sectors.

The revised model must distinguish different labor supply numbers across different households, distinguished at least by sector and search status. So let the utility functions and allocations take the form

\[ AU(c,T-a_H) \quad AU(c,T-a_L) \quad AU(c,T-S) \]

in the high- and low-productivity sectors and in search status mode, respectively. Here \( T \) is a common time endowment, \( a_H \) is hours for each worker in operational high-productivity sites, \( a_L \) is hours for each worker in operational low-productivity sites, and \( S \) is a fixed number of hours lost for those engaged in “search” or reallocation.

A priori every one is to be treated equally. Initially, then, one would just maximize the sum of all agents’ utilities. But as the economy evolves, people move around. In particular, \( \theta_i \) represents the fraction of households in the low-productivity sector who move, changing the count of the number of households in each sector. Still, one can also let \( \theta_i \) be the probability that it will be moved from the point of view of a household in the low-productivity sector. Then I have verified that the equal-weight Pareto optimum with utility over the explicit dynamic paths can be reduced into looking like the value function of a representative consumer. Namely,

\[
V(H_t, \sigma_t, A_t) = \max_{a_H, a_L, \theta_t} \left\{ (H_t - \sigma_t H_t) AU(c_i,T-a_H) + (1-H_t+\sigma_t H_t) \right\}
\]

\[ + \beta EV(H_{t+1}, \sigma_{t+1}, A_{t+1}) \]  

where

\[
\theta_t = \left\{ \frac{1}{(1-\theta_t) AU(c_i,T-a_L) + \theta_t AU(c_i,T-S)} \right\}
\]

\[
\frac{\text{movers}}{\text{fraction not moving conditioned on being in low sector}}
\]

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subject to law of motion (1) and to a resource constraint

\[ \begin{align*}
    c &= (1-\sigma_t)H_f(a_{H_t}) + (1-H_t+\sigma_tH_t)(1-\theta_t)f(a_{L_t}). \\
    \text{output from highs} & \quad \text{output from lows}
\end{align*} \]

A problem with the value function (5) as it is written is that moving is a lumpy decision variable. A household is to move or not, though one can see from dot expression in (6) that the random variable \( \theta_t \) smooths over this decision at the household level. Similarly, one is either in one sector or another, or in the search mode, and this may be "lumpy" because labor supply decisions vary over the three states. In short, the programming problem is not concave. But, this can be remedied by appropriate use of fractions or randomization.

In particular, let \( \pi_{H_t}(a,q,c) \) denote the fraction of households in the high-productivity sector who are to be assigned labor action \( a \), who are to suffer output \( q \) (recall this is random), and to receive consumption \( c \). Of course, output is determined by nature, probabilistically. That is, let \( \tilde{\pi}_{H_t}(q|a) \) denote the fraction of households in the high-productivity sector getting output \( q \) when action \( a \) is taken. To respect this one can impose a simple linear equality on endogenous choice variables \( \pi_{H_t}(a,q,c) \), namely:

\[ \sum_c \pi_H(a,q,c) = \text{Prob}(a,q) = \tilde{\pi}_{H_t}(q|a) \sum_q c \pi_H(a,q,c). \]  

(7)

For the low-productivity sector let \( \pi_L(a,q,c|m=0) \) denote the fraction of households assigned labor action \( a \), suffering output \( q \), and getting consumption \( c \), conditioned on not moving, \( m=0 \). Also, one can impose a constraint like (7) for \( \pi_L(a,q,c) \). Finally, let \( \pi_m(c|m=1) \) denote the fraction of movers getting consumption \( c \), and let \( \pi(m-1)=\theta \) denote the fraction of agents moving, the already familiar random variable \( \theta \). From the individual household's point of view, all the fractions represent probabilities.

With this notation the program for the determination of an equal weight Pareto optimum is to choose

\[ \pi_{H_t}(a,q,c), \pi_{L_t}(a,q,c|m=0), \pi_{m_t}(c|m=1), \theta_t=\pi(m-1). \]

To maximize:

\[ \begin{align*}
    V(H_t,\sigma_t,A_t) &= \{(H_t-\sigma_tH_t)[\Sigma_{a,q,c}AU(c,T-a)\pi_{H_t}(aro q,c)] \\
    &+ (1-H_t+\sigma_tH_t)[\cdot] + \beta EV[H_{t+1},\sigma_{t+1}, A_{t+1}] \}
\end{align*} \]

where

\[ \cdot = [\pi_t(m=0)\Sigma_{a,q,c}AU(c,T-a)\pi_{L_t}(a,q,c|m=0) + \pi_t(m=1) \Sigma_c AU(c,T-S)\pi_{m_t}(c|m-1)] \]

(8)
subject to (7) and its analogue for \( \pi_{Lk}(\cdot) \) and to a resource constraint, namely, consumption = output:

\[
H_t(1 - \sigma_t) \Sigma_{a,q,c} (c - q) \pi_{Ht}(a,q,c) + (1 - H_t + \sigma_t H_t) [\pi_t(m=0) \Sigma_{a,q,c} (c - q) \pi_{Lt}(a,q,c|m=0) + \pi_t(m=1) \Sigma_{c} \pi_{mt}(c|m=1)] = 0. \tag{9}
\]

A strategy for computing solutions to this program is suggested by what we have done before. Like \( H_t \) take on a finite number of values as before. Then take a guess for \( V \) on the right-hand side of (8). Next, fix decision variable \( \theta_t = \pi_t(m=1) \) at some arbitrary value. At this point, one can solve the program above as a linear program. That, among other things, is one of the virtues of the lottery notation. Finally, one can check all the others of a finite number of possible values for decision \( \theta_t \). Picking the best decision delivers a new guess for value function \( V \). One then should be able to iterate as before.

At this point we should ask a basic question: Do we really believe this prototype captures important features of the U.S. economy? That is, should we take solutions to the prototype seriously? Three objections come readily to mind.

First, the data is about employment in the manufacturing sector only, whereas in the United States there has been a trend away from manufacturing toward the service sector. This is more than apparent in inner-city neighborhoods like those of Chicago where unemployment has increased and incomes have decreased.

Second, job matching is modeled here as a simple one-period lag. There is no search per se and no variation in search unemployment. Nothing much about the search process feeds back to the individual decision problem. Frictions in the labor market, emphasized by Blanchard and Diamond (1989), are missing from the model (though one can begin to think up obvious remedies, while retaining the basic prototype).

Third, the model makes a strong prediction about consumption profiles in the population at a point: they are completely flat. A household's consumption is independent of which sector it is in. At most per capita consumption fluctuates over time with the state of the aggregate economy.

I am not inclined to believe this third feature of the model, the so-called full-insurance implication. A model with private information on labor effort seems much more appealing a priori, something that would make household consumption fluctuate with household income. This would give households an incentive to work hard by penalizing households who suffer low outputs. Indeed, a related prototype of Phelan (1989) is essentially the model here with one sector only and no aggre-
gate shocks. Essentially, one need only add an incentive constraint to induce households to take action $a$ over any other action $\bar{a}$, namely,

$$\sum_{q,c,w} \{U(c,t,a) + \beta w'\} \pi_r(a,q,c,w')$$

$$\geq \sum_{q,c,w} \{U(c,T-\bar{a}) + \beta w'\} \pi_r(\bar{a},q,c,w')[\pi(q|\bar{a})/\pi(q|a)]$$

(10)

for all actions $a$ and $\bar{a}$ in some set $A$, with $w'$ as expected utility from next period on.

Phelan’s model delivers a nontrivial, nonflat distribution of consumption and labor supply in the population. Related, it delivers time variation in consumption and labor efforts for each household, as households are rewarded or penalized for high and low outputs. In other words, it delivers a nontrivial level of gross employment changes and gross consumption changes at the microlevel even without aggregate shocks. Finally, average productivity is lower than in the analogue model with no incentive problem, in the model without (10).

A two-sector model with private information would force one to come to grips with some basic informational issues. One can imagine, for example, that labor effort remains unobserved as in the private information prototype above, but that the identity of one’s sector as well as aggregate shocks $\sigma$, and $A$, are fully observed. But one guesses for that information specification that consumption fluctuations would not be closely linked to sector-specific shocks $\sigma$. That is, being moved from one sector to another would not necessarily cause a household’s consumption to fluctuate beyond the effect that publicity observed variables have on everyone. Yet we see in PSID data the effect documented by Cochran (1989): workers who experience layoffs with protracted job search are those who experience diminished growth in consumptions.

If the identity or productivity of one’s plant or sector is private information, along with labor effort, then productivity shocks $\sigma$, would not be so well insured. Still, in the determination of an information-constrained optimum one would search ruthlessly for all random variables that might be revealing of these productivity shocks. Can anything much be inferred from firms “nearby,” distinguished by location or production line? Davis and Haltiwanger suggest the answer may be no, that most of the fluctuations at the establishment level are idiosyncratic. This could be one of their most important findings.

An extended private information prototype would guide one in how
to measure and quantify idiosyncratic and common components, would guide one in attempts to answer the question of whether there is any local, product line, or sector-specific information that is utilized or could be utilized to alleviate incentive problems. Indeed, we can ask whether observed fluctuations in employment and consumption are informationally constrained efficient. It is conceivable that the answer may be no, that unemployment insurance and other schemes might be modified in such a way as to reduce incentive problems. If so it seems this could increase average production and consumption, and reduce fluctuations in leisure and consumption. This possibility is something Pigou (1929) took seriously in his early treatment of industrial fluctuations. It is something one is led to naturally from consideration of the microunderpinnings for macroeconomic phenomena.

REFERENCES


Discussion

Martin Eichenbaum suggested that seasonal shocks were allocative shocks, so that the authors should leave them in the empirical work. He also wondered whether the model implies a linear VAR structure like the authors estimated.

Peter Diamond noted that the discrete sampling of data made it hard to infer flows of workers and vacancies from data on job creation and destruction. He also suggested that job creation need not lag behind job destruction if the allocation is in response to a positive productivity shock. Finally, he suggested similar government policies can have aggregate and allocative effects so that just differentiating between the types of shocks did not yield any implications about optimal government policy.

Ben Bernanke suggested that data on accessions and separations